Class XI Session 2023-24
Subject - Mathematics
Sample Question Paper - 9

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. If 5 cot θ = 4, then $\left(\frac{5\sin\theta - 3\cos\theta}{\sin\theta + 2\cos\theta}\right)$ = ? **[1]**

   a) 1 b) $\frac{3}{14}$

   c) $\frac{5}{14}$ d) $\frac{3}{4}$

2. If A = {(x, y) : $x^2 + y^2 = 5$} and B = {(x, y) : 2x = 5y}, then A ∩ B contains **[1]**

   a) two points b) one-point

   c) infinite points d) no point

3. If the variance of the data is V, then its S.D. is **[1]**

   a) $\pm\sqrt{V}$ b) $\sqrt{V}$

   c) V^2 d) - $\sqrt{V}$

4. The value of $\lim\limits_{x\to\infty} \frac{\sqrt{1+x^4}+(1+x^2)}{x^2}$ is: **[1]**

   a) 2 b) -1

   c) None of these d) 1

5. The point on the axis of y which is equidistant from (- 1, 2) and (3, 4) is **[1]**

   a) (0, 4) b) (4, 0)

   c) (5, 0) d) (0, 5)

6. Distance of the point (α, β, γ) from y-axis is **[1]**

   a) $\sqrt{\alpha^2 + \gamma^2}$ b) $|\beta| + |\gamma|$

c) $|\beta|$ d) β

7. $|z_1 + z_2| = |z_1| + |z_2|$ is possible if **[1]**

a) $z_2 = \overline{z_1}$ b) $\arg(z_1) = \arg(z_2)$

c) $|z_1| = |z_2|$ d) $z_2 = \frac{1}{z_1}$

8. For the post of 5 teachers, there are 23 applicants. 2 posts are reserved for SC candidates and there are 7 SC candidates among the applicants. In how many ways can the selection be made? **[1]**

a) 3920 b) 11760

c) None of these d) 5880

9. If f (x) = $\sqrt{1-x^2}$, x ∈ (0, 1), then f'(x), is equal to **[1]**

a) $\sqrt{1-x^2}$ b) $\sqrt{x^2-1}$

c) $\frac{1}{\sqrt{1-x^2}}$ d) $\frac{-x}{\sqrt{1-x^2}}$

10. A circular wire of radius 7 cm is cut and bent again into an arc of a circle of radius 12 cm. The angle subtended by the arc at the centre is **[1]**

a) 100^o b) 210^o

c) 50^o d) 60^o

11. If A = {x : x is a multiple of 3, x natural no., x < 30} and B = {x : x is a multiple of 5, x is natural no., x < 30} then A - B is **[1]**

a) {3, 6, 9, 12, 15, 18, 21, 24, 27, 30} b) {3, 6, 9, 12, 18, 21, 24, 27}

c) {3, 5, 6, 9, 10, 12, 15, 18, 20, 21, 25, 27, 30} d) {3, 6, 9, 12, 18, 21, 24, 27, 30}

12. $\{C_0 + 3C_1 + 5C_2 + ... + (2n + 1)C_n\} = ?$ **[1]**

a) None of these b) (n - 1) (n + 2)

c) $(n + 2) . 2^{n-1}$ d) $(n + 1)2^n$

13. The number 111111…………..1 (91 times) **[1]**

a) is not an odd number b) is an even number

c) is not a prime d) has a factor as 6

14. The solution set of the inequation: $\frac{2x-1}{3} - \frac{3x}{5} + 1 < 0, x \in W$ is: **[1]**

a) none of these b) x ∈ N

c) null set d) x ∈ W

15. If aN = {ax : x ∈ N}, then the set 3N ∩ 7N is **[1]**

a) 10N b) 7N

c) 21N d) 4N

16. If $\sin\theta = \frac{-4}{5}$, and θ lies in third quadrant then the value of $\cos\frac{\theta}{2}$ is **[1]**

a) $-\frac{1}{\sqrt{5}}$ b) $\frac{1}{5}$

c) $-\frac{1}{\sqrt{10}}$

d) $\frac{1}{\sqrt{10}}$

17. The value of $\lim_{x \to \pi/2} (\sec x - \tan x)$ is **[1]**

a) -1

b) 2

c) 1

d) 0

18. If the letters of the word KRISNA are arranged in all possible ways and these words are written out as in a dictionary, then the rank of the word KRISNA is **[1]**

a) None of these

b) 341

c) 324

d) 359

19. **Assertion (A):** The set A = {a, b, c, d, e, g} is finite set. **[1]**

Reason (R): The set B = {men living presently in different parts of the world} is finite set.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** The sum of first 6 terms of the GP 4, 16, 64, ... is equal to 5460. **[1]**

Reason (R): Sum of first n terms of the G.P is given by $S_n = \frac{a(r^n-1)}{r-1}$, where a = first term r = common ratio and |r| > 1.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Draw the graph of the signum function f(x) = $\begin{cases} \frac{|x|}{x} \text{ when } x \neq 0 \\ 0 \text{ when } x = 0 \end{cases}$ **[2]**

OR

Let A = {2, 3} and B = {3, 5}

i. Find $(A \times B)$ and the $n(A \times B)$.

ii. How many relations can be defined from A to B?

22. Differentiate the function from the first principle $\sqrt{\tan x}$. **[2]**

23. In a lottery of 50 tickets numbered 1 to 50, two tickets are drawn simultaneously. Find the probability that none of the tickets drawn has prime number. **[2]**

OR

A die is thrown twice. Each time the number appearing on it is recorded. Describe the following events:

i. A = Both numbers are odd.

ii. B = Both numbers are even.

iii. C = Sum of the numbers is less than 6

Also, find A ∪ B , A ∩ B, A ∪ C , A ∩ C. Which pairs of events are mutually exclusive?

24. For sets A, B and C using properties of sets, prove that: A - (B ∩ C) = (A - B) ∪ (A - C). **[2]**

25. Find the equation of the line which makes an angle of 30° with the positive direction of the x-axis and cuts off an intercept of 4 units with the negative direction of the y-axis. **[2]**

Section C

26. How many different words can be formed by using all the letters of the word ALLAHABAD? **[3]**

i. In how many of them, vowels occupy the even position?

ii. In how many of them, both L do not come together?

27. Find the coordinates of the point which is equidistant from the points A(a, 0, 0), B(0, b, 0), C(0, 0, c) and O(0, 0, 0). **[3]**

28. Show that the coefficient of the middle term in the expansion of$(1 + x)^{2n}$ is equal to the sum of the coefficients of middle terms in the expansion of $(1 + x)^{2n-1}$. **[3]**

OR

Find a if the coefficient of x^2 and x^3 in the expansion of $(3 + ax)^9$ are equal.

29. Evaluate: $\lim\limits_{x \to \infty} \frac{x}{\sqrt{4x^2+1}-1}$. **[3]**

OR

Differentiate sin (2x - 3) from first principle.

30. Evaluate: $\sum\limits_{k=1}^{11}\left(2+3^k\right)$ **[3]**

OR

Insert three geometric means between $\frac{1}{3}$ and 432.

31. If U = {2, 3, 5, 7, 9} is the universal set and A = {3, 7}, B = {2, 5, 7, 9}, then prove that: $(A \cap B)' = A' \cup B'$. **[3]**

Section D

32. While calculating the mean and variance of 10 readings, a student wrongly used the reading 52 for the correct reading 25. He obtained the mean and variance as 45 and 16 respectively. Find the correct mean and the variance. **[5]**

33. Find the equation of the hyperbola whose vertices are (- 8, -1) and (16, - 1) and focus is (17, - 1). **[5]**

OR

Referred to the principal axes as the axes of coordinates, find the equation of the hyperbola whose foci are at $(0, \pm \sqrt{10})$ and which passes through the point (2, 3).

34. Solve the following system of linear inequalities **[5]**

$\frac{4x}{3} - \frac{9}{4} < x + \frac{3}{4}$ and $\frac{7x-1}{3} - \frac{7x+2}{6} > x$.

35. Prove that: $\sin 5x = 5 \sin x - 20 \sin^3 x + 16 \sin^5 x$. **[5]**

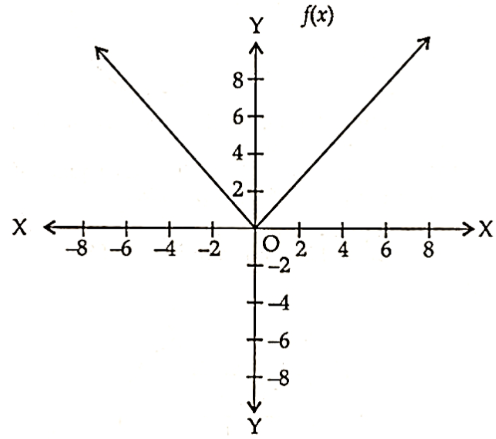
OR

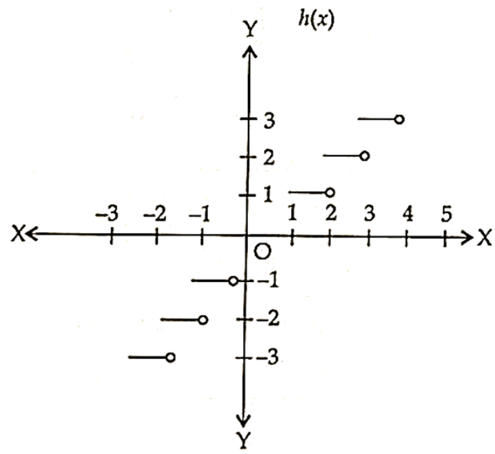
If $\sin\alpha=\frac{4}{5}$ and $\cos\beta = \frac{5}{13}$, prove that $\cos\frac{\alpha-\beta}{2} = \frac{8}{\sqrt{65}}$.

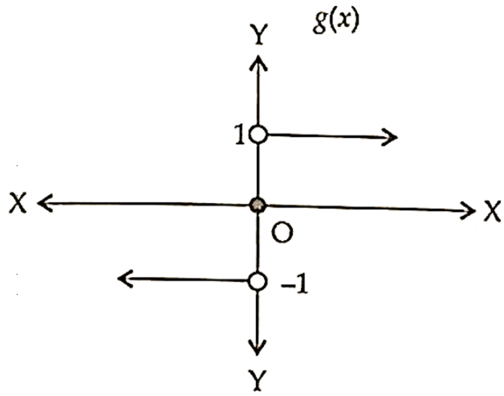
Section E

36. **Read the text carefully and answer the questions:** **[4]**

Consider the graphs of the functions f(x), h(x) and g(x).







(i) Find the range of h(x).

(ii) Find the domain of f(x).

(iii) Find the value of f(10).

OR

Find the range of g(x).

37. **Read the text carefully and answer the questions:** **[4]**

There are 4 red, 5 blue and 3 green marbles in a basket.

(i) If two marbles are picked at randomly, find the probability that both red marbles.

(ii) If three marbles are picked at randomly, find the probability that all green marbles.

(iii) If two marbles are picked at randomly then find the probability that both are not blue marbles.

OR

If three marbles are picked at randomly, then find the probability that atleast one of them is blue.

38. **Read the text carefully and answer the questions:** **[4]**

Two complex numbers $Z_1 = a + ib$ and $Z_2 = c + id$ are said to be equal, if a = c and b = d.

(i) If (x + iy)(2 - 3i) = 4 + i then find the value of (x, y).

(ii) If $\frac{(1+i)^2}{2-i}$ = x + iy, then find the value of x + y.

Solution

Section A

1. **(a)** 1

   Explanation: Given exp. $= \frac{(5-3\cot\theta)}{(1+2\cot\theta)} = \frac{\left(5-3\times\frac{4}{5}\right)}{\left(1+2\times\frac{4}{5}\right)} = \frac{(25-12)}{(5+8)} = \frac{13}{13}$ = 1.

2. **(a)** two points

   Explanation: From A, $x^2 + y^2 = 5$ and from B, $2x = 5y$

   Now, $2x = 5y \Rightarrow x = \frac{5}{2y}$

   $\therefore x^2 + y^2 = 5 \Rightarrow \left(\frac{5}{2y}\right)^2 + y^2 = 5$

   $\Rightarrow 29y^2 = 20 \Rightarrow y = \pm\sqrt{\frac{20}{29}}$

   $\Rightarrow 29y^2 = 20 \Rightarrow y = \pm\sqrt{\frac{20}{29}}$

   $\therefore x = \frac{5}{2}(\pm\sqrt{\frac{20}{29}})$

   $\therefore$ Possible ordered pairs = four

   But two ordered pair in which c is positive and y is negative will be rejected as it will not be satisifed by the equation in B.

   Therefore,

   $A \cap B$ contains 2 elements.

3. **(b)** $\sqrt{V}$

   Explanation: Standard deviation have the same units as the data but the variance is mean of the square of differences.

4. **(a)** 2

   Explanation: $\lim\limits_{x\to\infty} \frac{\sqrt{1+x^4}+(1+x^2)}{x^2}$

   $= \lim\limits_{x\to\infty} \sqrt{\frac{1}{x^4}+1} + \frac{1}{x^2} + 1$

   = 2

5. **(d)** (0, 5)

   Explanation: Let (0, y) be the point on Y-axis which is equidistant from the points (-1, 2) and (3, 4)

   By applying the distance formula,

   $(0 + 1)^2 + (y - 2)^2 = (3 - 0)^2 + (4 - y)^2$

   on simplifying we get

   $4y = 20$

   Therefore $y = 5$

   Hence the point on the y-axis is (0, 5)

6. **(a)** $\sqrt{\alpha^2 + \gamma^2}$

   Explanation: The foot of perpendicular from point $P(\alpha, \beta, \gamma)$ on y-axis is $Q(0, \beta, 0)$

   $\therefore$ Required distance, $PQ = \sqrt{(\alpha-0)^2 + (\beta-\beta)^2 + (\gamma-0)^2} = \sqrt{\alpha^2+\gamma^2}$

7. **(b)** arg (z_1) = arg (z_2)

   Explanation: Let $z_1 = r_1 (\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$

   Since $|z_1 + z_2| = |z_1| + |z_2|$

   $\Rightarrow z_1 + z_2 = r_1\cos\theta_1 + ir_1\sin\theta_1 + r_2\cos\theta_2 + ir_2\sin\theta_2$

   $\Rightarrow |z_1 + z_2|$

   $= \sqrt{r_1^2\cos^2\theta_1 + r_2^2\cos^2\theta_2 + 2r_1r_2\cos\theta_1\cos\theta_2 + r_1^2\sin^2\theta_1 + r_2^2\sin^2\theta_2 + 2r_1r_2\sin\theta_1\sin\theta_2}$

$= \sqrt{r_1^2 + r_2^2 + 2r_1r_2\cos(\theta_1 - \theta_2)}$

But $|z_1 + z_2| = |z_1| + |z_2|$

$\Rightarrow \sqrt{r_1^2 + r_2^2 + 2r_1r_2\cos(\theta_1 - \theta_2)} = r_1 + r_2$

Squaring both sides,

$\Rightarrow r_1^2 + r_2^2 + 2r_1r_2\cos(\theta_1 - \theta_2) = r_1^2 + r_2^2 + 2r_1r_2$

$\Rightarrow 2r_1r_2 - 2r_1r_2\cos(\theta_1 - \theta_2) = 0$

$\Rightarrow 1 - \cos(\theta_1 - \theta_2) = 0$

$\Rightarrow \cos(\theta_1 - \theta_2) = 1$

$\Rightarrow (\theta_1 - \theta_2) = 0$

$\Rightarrow \theta_1 = \theta_2$

$\therefore \arg(z_1) = \arg(z_2)$

8.

(b) 11760

Explanation: We have to select 2 posts out of 7 SC and 3 posts out of 16.

Required number of ways $= ({}^7C_2 \times {}^{16}C_3) = \left(\frac{7\times6}{2} \times \frac{16\times15\times14}{3\times2\times1}\right)$ = 11760.

9.

(d) $\frac{-x}{\sqrt{1-x^2}}$

Explanation: f(x) = $\sqrt{1-x^2}$

f'(x) = $\frac{1}{2\sqrt{1-x^2}} - 2x = \frac{-x}{\sqrt{1-x^2}}$

10.

(b) 210^o

Explanation: Here, radius of circular wire is r = 7 cm

So, length of wire = $2 \times \pi \times r$

$= 2 \times \pi \times 7$

$= 14 \times \pi$

Wire is cut and bent again into an arc of a circle of radius 12 cm.

So, length of arc=length of wire=$14\times \pi$

We know, angle subtended by the arc is given by,

$\theta = \frac{\text{length of arc}}{\text{radius}}$

$= \frac{14\pi}{12}$

$= \frac{14\pi}{12} \times \frac{180^\circ}{\pi}$

$= 210^o$

11.

(b) {3, 6, 9, 12, 18, 21, 24, 27}

Explanation: Since set B represent multiple of 5 so from Set A common multiple of 3 and 5 are excluded.

12.

(d) $(n + 1)2^n$

Explanation: We have, $C_0 + 3C_1 + 5C_2 + ...+ (2n + 1)C_n$

$= (C_0 + C_1 + C_2 + ... + C_n) + 2(C_1 + 2C_2 + ... + nC_n)$

$= 2^n + 2(n \cdot 2^{n-1}) = (n + 1) \cdot 2^n$

13.

(c) is not a prime

Explanation: 111...111(91times) can be expressed as:-

$\frac{1}{9}(10^{91} - 1) \Rightarrow \frac{1}{9}(10^7 - 1) \times x$

$\Rightarrow 1111111 \times x$

Where $x = (10^7)^{12} + (10^7)^{11} + ... + 1$

14.

(c) null set

Explanation: $\frac{2x-1}{3} - \frac{3x}{5} + 1 < 0$

$\Rightarrow 15 \cdot \frac{2x-1}{3} - 15 \cdot \frac{3x}{5} + 15 < 0$ [Multiply the inequality throughout by the L.C.M]

$\Rightarrow$ 5(2x - 1) -3(3x) + 15 < 0

$\Rightarrow$ 10x - 5 - 9x + 15 < 0

$\Rightarrow$ x + 10 < 0

$\Rightarrow$ x < -10, but given x $\in$ W

Hence the solution set will be null set.

15.

(c) 21N

Explanation: Here 3N = {3, 6, 9,...} and 7N = {7, 14, 21,...}

Hence 3N $\cap$ 7N = {21, 42, ...} = {21x : x $\in$ N} = 21N

16. **(a)** $-\frac{1}{\sqrt{5}}$

Explanation: Given that $\sin\theta = \frac{-4}{5}$ and θ lies in third quadrant.

$\cos\theta = \sqrt{1-\sin^2\theta}$

= $\sqrt{1-\left(\frac{-4}{5}\right)^2}$

= $\sqrt{1-\frac{16}{25}}$

= $\sqrt{\frac{9}{25}}$

= $\pm\frac{3}{5}$

$\Rightarrow \cos\theta = -\frac{3}{5}$ since θ lies in third quadrant

$\cos\theta = 2\cos^2\frac{\theta}{2} - 1$

$\Rightarrow 2\cos^2\frac{\theta}{2} = 1 - \frac{3}{5}$ = $\frac{2}{5}$

$\Rightarrow \cos^2\frac{\theta}{2} = \frac{1}{5}$

$\Rightarrow \cos\frac{\theta}{2} = \pm\frac{1}{\sqrt{5}}$

$\Rightarrow \cos\frac{\theta}{2} = -\frac{1}{\sqrt{5}}$ (since $\frac{\theta}{2}$ lies in second quadrant)

17.

(d) 0

Explanation: $\lim\limits_{x\to\frac{\pi}{2}}(\sec x - \tan x)$

$= \lim\limits_{h\to0}\left(\sec\left(\frac{\pi}{2}-h\right) - \tan\left(\frac{\pi}{2}-h\right)\right)$

$= \lim\limits_{h\to0}(\text{cosec h} - \cot h)$

$= \lim\limits_{h\to0}\frac{1-\cos h}{\sin h}$

$= \lim\limits_{h\to0}\frac{2\sin^2\frac{h}{2}}{\sin h}$

$= \lim\limits_{h\to0}\frac{2\sin^2\frac{h}{2}}{2\sin\frac{h}{2}\cos\frac{h}{2}}$

$= \lim\limits_{h\to0}\tan\frac{h}{2}$

= 0

18.

(c) 324

Explanation: When arranged alphabetically, the letters of the word KRISNA are A, I, K, N, R and S.

Number of words that will be formed with A as the first letter = Number of arrangements of the remaining 5 letters = 5!

Number of words that will be formed with I as the first letter = Number of arrangements of the remaining 5 letters = 5!

$\therefore$ The number of words beginning with KA = Number of arrangements of the remaining 4 letters = 4!

The number of words starting with KI = Number of arrangements of the remaining 4 letters = 4!

Alphabetically, the next letter will be KR. Number of words starting with KR followed by A, i.e. KRA = Number of arrangements of the remaining 3 letters = 3!

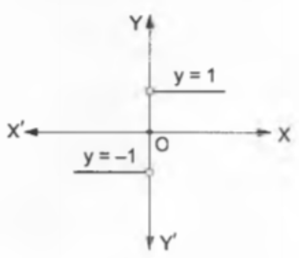
Number of words starting with KRI followed by A, i.e. KRIA = Number of arrangements of the remaining 2 letter = 2!
Number of words starting with KRI followed by N, i.e. KRIN = Number of arrangements of the remaining 2 letter = 2!
The first word beginning with KRIS is the word KRISAN and the next word is KRISNA.
∴ Rank of the word KRISNA = 5! + 5! + 4! + 4! + 3! + 2! + 2! + 2 = 324

19.

(b) Both A and R are true but R is not the correct explanation of A.
Explanation: Assertion: We know that, a set which is empty or consists of a definite number of elements, is called finite, otherwise the set is called infinite. Since, set A contains finite number of elements. So, it is a finite set.
Reason: We do not know the number of elements in B, but it is some natural number. So, B is also finite.

20. **(a)** Both A and R are true and R is the correct explanation of A.
Explanation: Assertion: Given GP 4, 16, 64, ...
∴ a = 4, r = $\frac{16}{4}$ = 4 > 1
∴ S_6 = $\frac{4\left((4)^6-1\right)}{4-1} = \frac{4(4095)}{3}$ = 5460
Hence, Assertion and Reason both are true and Reason is the correct explanation of Assertion.

Section B

21. 

Clearly, (0, 0) is a point on the graph. Now, when x > 0, we have |x| = x, and so in this case, we have, f(x) = 1, i.e., f(x) = 1 for all values of x >0.
And, when x < 0, we have |x| = -x
therefore, f(x) = -1 for all values of x < 0
Hence the graph may be drawn, as shown in the adjoining figure.
Clearly, the function is broken (i.e., it is discontinuous) at each of the points x = -1, 0 and 1.

OR

Here we have, A = {2, 3} and B = {3, 5}

i. To find: $(A \times B)$ and $n(A \times B)$
$(A \times B)$ = {(2, 3), (2, 5), (3, 3), (3, 5)}
Thus, $n(A \times B)$ = 4
ii. As we know that: $(A \times B) = 2 \times 2 = 4$
So, the total number of relations can be defined from A to B
= 2^4 = 16

22. Let $y = \sqrt{\tan x}$
Then, $y + \delta y = \sqrt{\tan(x + \delta x)}$
$\Rightarrow \delta y = \sqrt{\tan(x + \delta x)} - \sqrt{\tan x}$
$\Rightarrow \frac{\delta y}{\delta x} = \frac{\sqrt{\tan(x+\delta x)}-\sqrt{\tan x}}{\delta x}$
$\Rightarrow \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\delta y}{\delta x}$
$= \lim_{\delta x \to 0} \frac{\sqrt{\tan(x+\delta x)}-\sqrt{\tan x}}{\delta x}$
$= \lim_{\delta x \to 0} \left\{\frac{\sqrt{\tan(x+\delta x)}-\sqrt{\tan x}}{\delta x} \times \frac{\sqrt{\tan(x+\delta x)}+\sqrt{\tan x}}{\sqrt{\tan(x+\delta x)}+\sqrt{\tan x}}\right\}$
$= \lim_{\delta x \to 0} \frac{\tan(x+\delta x)-\tan x}{\delta x[\sqrt{\tan(x+\delta x)}+\sqrt{\tan x}]}$
$= \lim_{\delta x \to 0} \frac{\left\{\frac{\sin(x+\delta x)}{\cos(x+\delta x)} - \frac{\sin x}{\cos x}\right\}}{\delta x[\sqrt{\tan(x+\delta x)}+\sqrt{\tan x}]}$

$= \lim_{\delta x \to 0} \frac{\sin(x+\delta x)\cos x - \cos(x+\delta x)\sin x}{\cos(x+\delta x)\cos x \cdot \delta x \cdot [\sqrt{\tan(x+\delta x)} + \sqrt{\tan x}]}$

$= \lim_{\delta x \to 0} \frac{\sin(x+\delta x - x)}{\cos(x+\delta x) \cdot \cos x \cdot \delta x \cdot (\sqrt{\tan(x+\delta x)} + \sqrt{\tan x})}$ [using sin(A-B)=sinA cos B-cosA sin B]

$= \frac{1}{\cos x} \cdot \lim_{\delta x \to 0} \frac{1}{\cos(x+\delta x)} \cdot \lim_{\delta x \to 0} \frac{\sin \delta x}{\delta x}$

$. \lim_{\delta x \to 0} \frac{1}{(\sqrt{\tan(x+\delta x)} + \sqrt{\tan x})}$

$= \left(\frac{1}{\cos x} \cdot \frac{1}{\cos x} \cdot 1 \cdot \frac{1}{2\sqrt{\tan x}}\right) = \frac{\sec^2 x}{2\sqrt{\tan x}}$

Hence, $\frac{d}{dx}(\sqrt{\tan x}) = \frac{\sec^2 x}{2\sqrt{\tan x}}$

23. We have to find the probability that none of the tickets drawn has a prime number.

Out of 50 tickets, 2 tickets can be drawn in $^{50}C_2$ ways

So, the total number of elementary events $^{50}C_2 = 1225$

Number of non-primes from 1 to 50 = 50 - 15 = 35.

Out of these 35 numbers 2 can be selected in $^{35}C_2$ ways.

$\therefore$ Favourable number of elementary events = $^{35}C_2 = 595$

So, required probability = $\frac{595}{1225} = \frac{17}{35}$

OR

We have given that

A dice is thrown twice. And each time number appearing on it is recorded.

We have to find:

i. A = Both numbers are odd.
ii. B = Both numbers are even.
iii. C = Sum of the numbers is less than 6

Explanation: when the dice is thrown twice then the number of sample spaces are $6^2 = 36$

Now,

The possibility both odd numbers are:

A = {(1, 1), (1, 3), (1, 5), (3, 1), (3, 3), (3, 5), (5, 1), (5, 3), (5, 5)}

Since, Possibility of both even numbers are:

B = {(2, 2)(2, 4)(2, 6)(4, 2)(4, 4)(4, 6)(6, 2)(6, 4)(6, 6)}

And, Possible outcome of sum of the numbers is less than 6

C = {(1, 1)(1, 2)(1, 3)(1, 4)(2, 1)(2, 2)(2, 3)(3, 1)(3, 2)(4, 1)}

Therefore,

(A ∪B) = {(1, 1), (1, 3), (1, 5), (3, 1), (3, 3), (3, 5), (5, 1), (5, 3), (5, 5) (2, 2)(2, 4)(2, 6)(4, 2)(4, 4)(4, 6)(6, 2)(6, 4)(6, 6)}

(A∩B) = {Φ}

(A∪C) = {(1, 1), (1, 3), (1, 5), (3, 1), (3, 3), (3, 5), (5, 1), (5, 3), (5, 5) (1, 2)(1, 4)(2, 1)(2, 2)(2, 3)(3, 1)(3, 2)(4, 1)}

(A∩C) = {(1, 1)(1, 3)(3, 1)}

Hence, (A∩B) = ∅ and (A∩C) ≠ ∅, A and B are mutually exclusive, but A and C are not.

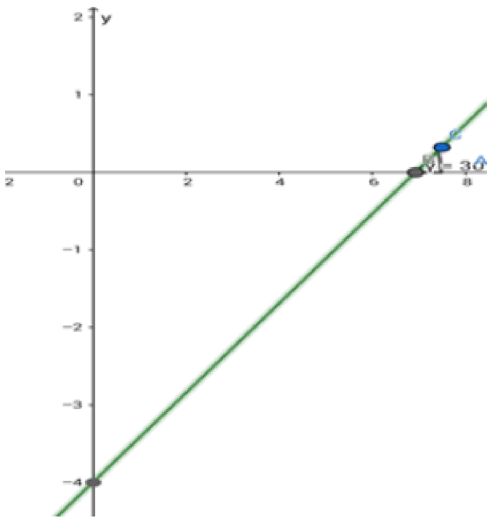
24. We have L.H.S , A - (B ∩ C) = A ∩ (B ∩ C)' [∵ X - Y = X ∩ Y']

= A ∩ (B' ∪ C') [∵ (B ∩ C)' = B' ∪ C']

= (A ∩ B') ∪ (A ∩ C') [∵ ∩ is distributive over ∪]

= (A - B) ∪ (A - C) = R.H.S

Hence proved.

25.  Here,it is given:The given line makes an angle of 30° with the x-axis. The y-intercept = - 4

Therefore, the slope of the line is m $= \tan\theta = \tan 30^\circ = 1/\sqrt{3}$

Formula to be used: y = mx + c where m is the slope of the line and c is the y-intercept.

Therefore,the required equation of the line is y $= \frac{1}{\sqrt{3}}x - 4$

Or, $\sqrt{3y} = x - 4\sqrt{3}$ i.e. $x - \sqrt{3}y = 4\sqrt{3}$

Section C

26. In a word ALLAHABAD, we have

Letters	A	L	H	B	D	Total
Number	4	2	1	1	1	9

So, the total number of words = $\frac{9!}{4!2!} = \frac{9\times8\times7\times6\times5}{2\times1}$ = 7560

i. There are 4 vowels and all are alike i.e., 4 A's.

Also, there are 4 even places which are 2nd, 4th, 6th and 8th. So, these 4 even places can be occupied by 4 vowels in $\frac{4!}{4!}$ = 1 way. Now, we are left with 5 places in which 5 letters, of which two are alike (2 L's) and other distinct, can be arranged in $\frac{5!}{2!}$ ways.

Hence, the total number of words in which vowels occupy the even places $= \frac{5!}{2!} \times \frac{4!}{4!} = \frac{5!}{2!}$ = 60

ii. Considering both L together and treating them as one letter. We have,

Letters	A	LL	H	B	D	Total
Number	4	1	1	1	1	8

Then, 8 letters can be arranged in $\frac{8!}{4!}$ ways.

So, the number of words in which both L come together = $\frac{8!}{4!}$ = 8 × 7 × 6 × 5 = 1680

Hence, the number of words in which both L do not come together

= Total number of words - Number of words in which both L come together

= 7560 - 1680 = 5880

Hence, the total number of words in which both L do not come together is 5880

27. Consider, D(x,y,z) point equidistant from points A(a, 0, 0), B(0, b, 0), C(0, 0, c) and O(0, 0, 0).

$\therefore$ AD = OD

$\sqrt{(x-a)^2 + (y-0)^2 + (z-0)^2} = \sqrt{(x-0)^2 + (y-0)^2 + (z-0)^2}$

Squaring both sides,

$(x - a)^2 + (y - 0)^2 + (z - 0)^2 = (x - 0)^2 + (y - 0)^2 + (z - 0)^2$

$x^2 + 2ax + a^2 + y^2 + z^2 = x^2 + y^2 + z^2$

a(2x - a) = 0

as a $\neq$ 0.

X = a/2

$\therefore$ BD = OD

$\sqrt{(x-a)^2 + (y-0)^2 + (z-0)^2} = \sqrt{(x-0)^2 + (y-0)^2 + (z-0)^2}$

Squaring both sides,

(x - 0)^2 + (y - b)^2 + (z - 0)^2 = (x - 0)^2 + (y - 0)^2 + (z - 0)^2

$x^2 + y^2 + 2by + b^2 + z^2 = x^2 + y^2 + z^2$

b(2y - b) = 0

as b $\neq$ 0.

y= b/2

$\therefore$ CD = OD

$\sqrt{(x-0)^2+(y-0)^2+(z-c)^2} = \sqrt{(x-0)^2+(y-0)^2+(z-0)^2}$

Squaring both sides,

(x - 0)^2 + (y - 0)^2 + (z - c)^2 = (x - 0)^2 + (y - 0)^2 + (z - 0)^2

$x^2 + y^2 + z^2 + 2cz + c^2 = x^2 + y^2 + z^2$

c(2z - c) = 0

as c $\neq$ 0.

z= c/2

Therefore, the pint D(a/2, b/2, c/2) is equidistant to points A(a, 0, 0), B(0, b, 0), C(0, 0, c) and O(0, 0, 0)

28. As discussed in the previous example, the middle term in the expansion of $(1 + x)^{2n}$ is given by $T_{n+1} = {}^{2n}C_n x^n$

So, the coefficient of the middle term in the expansion of $(1 + x)^{2n}$ is ${}^{2n}C_n$.

Now, consider the expansion of $(1 + x)^{2n-1}$ Here, the index (2n-1) is odd.

So, $\left(\frac{(2n-1)+1}{2}\right)^{th}$ and $\left(\frac{(2n-1)+1}{2}+1\right)^{th}$ i.e., n^{th} and $(n + 1)^{th}$ terms are middle terms.

Now, $T_n = T_{(n-1)+1}, = {}^{2n-1}C_{n-1}(1)^{(2n-1)-(n-1)}x^{n-1} = {}^{2n-1}C_{n-1}x^{n-1}$

and, $T_{n+1} = {}^{2n-1}C_n(1)^{(2n-1)-n}x^n = {}^{2n-1}C_n x^n$

So, the coefficients of two middle terms in the expansion of $(1 + x)^{2n-1}$ are ${}^{2n-1}C_{n-1}$ and ${}^{2n-1}C_n$.

$\therefore$ Sum of these coefficients = ${}^{2n-1}C_{n-1} + {}^{2n-1}C_n$

= ${}^{(2n-1)+1}C_n$ $[\because {}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r]$

= ${}^{2n}C_n$

= Coefficient of middle term in the expansion of $(1 + x)^{2n}$.

OR

Here $(3 + ax)^9 = {}^9C_0(3)^9 + {}^9C_1(3)^8(ax) + {}^9C_2(3)^7(ax)^2 + {}^9C_3(3)^6(ax)^3 + \ldots$

$= {}^9C_0(3)^9 + {}^9C_1(3)^8 \cdot a \cdot x + {}^9C_2(3)^7(a)^2 \cdot x^2 + {}^9C_3(3)^6 \cdot a^3x^3 + \ldots$

$\therefore$ Coefficient of $x^2 = {}^9C_2(3)^7a^2$

Coefficient of $x^3 = {}^9C_3(3)^6a^3$

It is given that

${}^9C_2(3)^7a^2 = {}^9C_3(3)^6a^3 \Rightarrow 36 \cdot 3^7a^2 = 84 \cdot 3^6 \cdot a^3$

$\Rightarrow a = \frac{36 \cdot 3^7}{84 \cdot 3^6} = \frac{108}{84} = \frac{9}{7}$.

29. We have to evaluate, $\lim_{x\to\infty}\left[\frac{x}{\sqrt{4x^2+1}-1}\right]$

Rationalising the denominator:

$\lim_{x\to\infty}\left[\frac{x}{(\sqrt{4x^2+1}-1)} \frac{(\sqrt{4x^2+1}+1)}{(\sqrt{4x^2+1}+1)}\right]$

$= \lim_{x\to\infty}\left[\frac{x(\sqrt{4x^2+1}+1)}{4x^2+1-1}\right]$

$= \lim_{x\to\infty}\left[\frac{\sqrt{4x^2+1}+1}{4x}\right]$

Dividing the numerator and the denominator by x:

$\lim_{x\to\infty}\left[\frac{\frac{\sqrt{4x^2+1}}{x}+\frac{1}{x}}{4}\right]$

$= \lim_{x\to\infty}\left[\frac{\sqrt{\frac{4x^2+1}{x^2}}+\frac{1}{x}}{4}\right]$

$= \lim_{x\to\infty}\left[\frac{\sqrt{4+\frac{1}{x^2}}+\frac{1}{x}}{4}\right]$

$x \to \infty$

$\therefore \frac{1}{x}, \frac{1}{x^2} \to 0$

$= \frac{\sqrt{4}}{4}$

$= \frac{2}{4}$

$= \frac{1}{2}$

OR

We need to find derivative of f(x) = sin (2x – 3)

Derivative of a function f(x) is given by f'(x) = $\lim_{h\to0} = \frac{f(x+h)-f(x)}{h}$ {where h is a very small positive number}

$\therefore$ derivative of f(x) = sin (2x – 3) is given as f'(x) = $\lim_{h\to0} = \frac{f(x+h)-f(x)}{h}$

$\Rightarrow f'(x) = \lim_{h\to0}\frac{\sin(2(x+h)-3)-\sin(2x-3)}{h}$

Use: sin A – sin B = 2 cos $\left(\frac{(A+B)}{2}\right)$ sin $\left(\frac{(A-B)}{2}\right)$

$\Rightarrow f'(x) = \lim_{h\to0}\frac{2\cos\left(\frac{4x-6+2h}{2}\right)\sin\left(\frac{2h}{2}\right)}{h}$

$\Rightarrow f'(x) = 2\lim_{h\to0}\frac{\cos(2x-3+h)\sin(h)}{h}$

$\Rightarrow f'(x) = 2\lim_{h\to0}\frac{\sin(h)}{h} \times \lim_{h\to0}$ cos (2x - 3 + h)

Use the formula $\lim_{h\to0}\frac{\sin(h)}{h}$ = 1

$\therefore$ f'(x) = 2 $\times \lim_{h\to0}$ cos (2x - 3 + h)

$\therefore$ f'(x) = 2 cos (2x – 3 + 0) = 2cos (2x – 3)

30. Given: $\sum_{k=1}^{11}\left(2+3^k\right)$

= $(2 + 3^1) + (2 + 3^2) + (2 + 3^3) + (2 + 3^{11})$

= (2 + 2 + 2 +........11 times) + $(3 + 3^2 + 3^3 + ... + 3^{11})$

= 22 + $(3 + 3^2 + 3^3 + ... + 3^{11})$ ……….(i)

Here 3, $3^2, 3^3$ .... ,3^{11} is in G.P.

$\therefore$a = 3 and r = $\frac{3^2}{3} = 3$

$S_n = \frac{3(3^{11}-1)}{3-1} = \frac{3}{2}(3^{11}-1)$

Putting the value of S_n in eq. (i), we get $\sum_{k=1}^{11}\left(2+3^k\right) = 22 + \frac{3}{2}(3^{11}-1)$

OR

Given: the numbers $\frac{1}{3}$ and 432.

By using Formula, r = $\left(\frac{b}{a}\right)^{\frac{1}{n+1}}$ where n is the number of geometric mean.

Suppose G_1, G_2 and G_3 be the three geometric mean

Then r = $\left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

$\Rightarrow$ r = $\left(\frac{432}{\frac{1}{3}}\right)^{\frac{1}{3+1}}$

$\Rightarrow$ r = $\left(\frac{432\times2}{1}\right)^{\frac{1}{4}}$ $\Rightarrow$ r = 6

G_1 = ar = $\frac{1}{3} \times 6$ = 2

G_2 = ar^2 = $\frac{1}{3} \times 6^2$ = 12

G_3 = ar^3 = $\frac{1}{3} \times 6^3 = \frac{1}{3} \times 216$ = 72

Therefore, three geometric mean between $\frac{1}{3}$ and 432 are 2, 12 and 72.

31. We have, (A ∩ B) = {x : x ∈ A and x ∈ B}

= {7}

(A ∩ B)' means Complement of (A ∩ B) with respect to universal set U.

Therefore, (A ∩ B)' = U - (A ∩ B)

U - (A ∩ B)' is defined as {x ∈ U : x ∉ (A ∩ B)'}

U = {2, 3, 5, 7, 9}

(A ∩ B)' = {7}

U - (A ∩ B)' = {2, 3, 5, 9}

A' means Complement of A with respect to universal set U.

Therefore, A' = U - A

U - A is defined as {x ∈ U : x ∉ A}

U = {2, 3, 5, 7, 9}

A = {3, 7}

A' = {2, 5, 9}

B' means Complement of B with respect to universal set U.

Therefore, B' = U - B

U - B is defined as {x ∈ U : x ∉ B}

U = {2, 3, 5, 7, 9}

B = {2, 5, 7, 9}.

B' = {3}

A' ∪ B' = {x: x ∈ A or x ∈ B }

= {2, 3, 5, 9}

Hence verified.

Section D

32. To find: the correct mean and the variance.

As per given criteria,

Number of reading, n=10

Mean of the given readings before correction, $\bar{x} = 45$

But we know,

$\bar{x} = \frac{\sum x_i}{n}$

Substituting the corresponding values, we get

$45 = \frac{\sum x_i}{10}$

$\Rightarrow \sum x_i = 45 \times 10 = 450$

It is said one reading 25 was wrongly taken as 52,

So $\sum x_i = 450 - 52 + 25 = 423$

So the correct mean after correction is

$\bar{x} = \frac{\sum x_i}{n} = \frac{423}{10} = 42.3$

Also given the variance of the 10 readings is 16 before correction,

i.e., σ^2=16

But we know

$\sigma^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2$

Substituting the corresponding values, we get

$16 = \frac{\sum x_i^2}{10} - (45)^2$

$\Rightarrow 16 = \frac{\sum x_i^2}{10} - 2025$

$\Rightarrow 16 + 2025 = \frac{\sum x_i^2}{10}$

$\Rightarrow \frac{\Sigma x_i^2}{10} = 2041$

$\Rightarrow \sum x_i^2 = 20410$

It is said one reading 25 was wrongly taken as 52, so

$\Rightarrow \sum x_i^2 = 20410 - (52)^2 + (25)^2$

$\Rightarrow \sum x_i^2 = 20410 - 2704 + 625$

$\Rightarrow \sum x_i^2 = 18331$

So the correct variance after correction is

$\sigma^2 = \frac{18331}{10} - \left(\frac{423}{10}\right)^2$

σ^2 = 1833.1 - $(42.3)^2$ = 1833.1 - 1789.29

σ^2 = 43.81

Hence the corrected mean and variance is 42.3 and 43.81 respectively.

33. The centre of the hyperbola is the mid-point of the line joining the two vertices.

So, the coordinates of the centre are $\left(\frac{16-8}{2}, \frac{-1-1}{2}\right)$ i.e., (4, -1).

Let 2a and 2b be the length of transverse and conjugate axes and let e be the eccentricity. Then, the equation of the hyperbola is

$\frac{(x-4)^2}{a^2} - \frac{(y+1)^2}{b^2} = 1$ ....(i)

Now, The distance between two vertices = 2a

$\therefore \sqrt{(16+8)^2+(-1+1)^2}$ = 2a [$\because$ vertices = (-8, -1) and (16, -1)]

$\Rightarrow$ 24 = 2a

$\Rightarrow$ a = 12

$\Rightarrow$ a^2 = 144

and, the distance between the focus and vertex is = ae - a

$\therefore \sqrt{(17-16)^2+(-1+1)^2}$ = ae - a

$\Rightarrow \sqrt{1^2}$ = ae - a

$\Rightarrow$ ae - a = 1

$\Rightarrow$ 12 $\times$ e - 12 = 1

$\Rightarrow$ 12e = 1 + 12

$\Rightarrow$ e = $\frac{13}{12}$

$\Rightarrow$ $e^2 = \frac{169}{144}$

Now,

$b^2 = a^2 (e^2 - 1)$

= $(12)^2 \left(\frac{169}{144} - 1\right)$

= 144 $\times \left(\frac{169-144}{144}\right)$

= 144 $\times \frac{25}{144}$

= 25

Putting a^2 = 144 and b^2 = 25 in equation (1), we get

$\frac{(x-4)^2}{144} - \frac{(y+1)^2}{25} = 1$

$\Rightarrow \frac{25(x-4)^2-144(y+1)^2}{3600} = 1$

$\Rightarrow 25\left[x^2+16-8x\right] - 144\left[y^2+1+2y\right] = 3600$

$\Rightarrow$ $25x^2 + 400 - 200x - 144y^2 - 144 - 288y = 3600$

$\Rightarrow$ $25x^2 - 144y^2 - 200x - 288y + 256 = 3600$

$\Rightarrow$ $25x^2 - 144y^2 - 200x - 288y - 3344 = 0$

This is the equation of the required hyperbola.

OR

Since the vertices are on y-axis, so let the equation of the required hyperbola be

$\frac{x^2}{a^2} - \frac{y^2}{b^2}$ = - 1 .....(i)

It passes through (2, 3).

$\therefore \frac{4}{a^2} - \frac{9}{b^2}$ = -1

$\Rightarrow \frac{4}{b^2(e^2-1)} - \frac{9}{b^2}$ = - 1 ...[$\because a^2 = b^2 (e^2 - 1)$]

$\Rightarrow \frac{4}{b^2e^2-b^2} - \frac{9}{b^2}$ = - 1 ...(ii)

The coordinates of foci are given to be (0, $\pm\sqrt{10}$).

$\therefore$ be = $\sqrt{10} \Rightarrow b^2 e^2$ = 10 ...(iii)

From (ii) and (iii), we get

$\frac{4}{10-b^2} - \frac{9}{b^2}$ = -1

$\Rightarrow$ $4b^2 - 9(10 - b^2) = - b^2(10 - b^2)$

$\Rightarrow 13b^2 - 90 = -10b^2 + b^4$

$\Rightarrow b^4 - 23b^2 + 90 = 0 \Rightarrow (b^2 - 18)(b^2 - 5) = 0 \Rightarrow b^2 = 18$ or, $b^2 = 5$

Now, $a^2 = b^2(e^2 - 1) \Rightarrow a^2 = (be)^2 - b^2 \Rightarrow a^2 = 10 - b^2$ $[\because be = \sqrt{10}]$

If $b^2 = 18$, then $a^2 = 10 - b^2 \Rightarrow a^2 = 10 - 18 = -8$, which is not possible.

$\therefore b^2 = 5$ and hence $a^2 = 10 - b^2 \Rightarrow a^2 = 10 - 5 = 5$.

Substituting the values of a^2 and b^2 in (i), we obtain

$\frac{x^2}{5} - \frac{y^2}{5} = -1$ i.e. $x^2 - y^2 = -5$ as the equation of the required hyperbola.

34. We have, $\frac{4x}{3} - \frac{9}{4} < x + \frac{3}{4}$ ... (i)

and $\frac{7x-1}{3} - \frac{7x+2}{6} > x$ ... (ii)

From inequality (i), we get

$\frac{4x}{3} - \frac{9}{4} < x + \frac{3}{4} \Rightarrow \frac{16x-27}{12} < \frac{4x+3}{4}$

$\Rightarrow 16x - 27 < 12x + 9$ [multiplying both sides by 12]

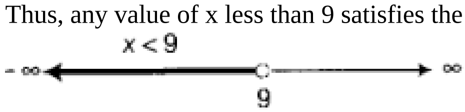
$\Rightarrow 16x - 27 + 27 < 12x + 9 + 27$ [adding 27 on both sides]

$\Rightarrow 16x < 12x + 36$

$\Rightarrow 16x - 12x < 12x + 36 - 12x$ [subtracting 12x from bot sides]

$\Rightarrow 4x < 36 \Rightarrow x < 9$ [dividing both sides by 4]

Thus, any value of x less than 9 satisfies the inequality. So, the solution of inequality (i) is given by $x \in (-\infty, 9)$



From inequality (ii) we get,

$\frac{7x-1}{3} - \frac{7x+2}{6} > x \Rightarrow \frac{14x-2-7x-2}{6} > x$

$\Rightarrow 7x - 4 > 6x$ [multiplying by 6 on both sides]

$\Rightarrow 7x - 4 + 4 > 6x + 4$ [adding 4 on both sides]

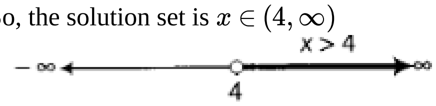
$\Rightarrow 7x > 6x + 4$

$\Rightarrow 7x - 6x > 6x + 4 - 6x$ [subtracting 6x from both sides]

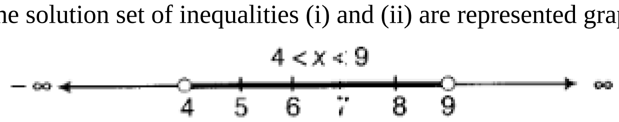
$\therefore x > 4$

Thus, any value of x greater than 4 satisfies the inequality.

So, the solution set is $x \in (4, \infty)$



The solution set of inequalities (i) and (ii) are represented graphically on number line as given below:



Clearly, the common value of x lie between 4 and 9.

Hence, the solution of the given system is, $4 < x < 9$ i.e., $x \in (4, 9)$

35. We have to prove that $\sin 5x = 5 \sin x - 20 \sin^3 x + 16 \sin^5 x$.

Let us consider LHS = sin 5x

$\sin 5x = \sin(3x + 2x)$

But we know,

$\sin(x + y) = \sin x \cos y + \cos x \sin y$ ... (i)

$\Rightarrow \sin 5x = \sin 3x \cos 2x + \cos 3x \sin 2x$

$\Rightarrow \sin 5x = \sin (2x + x) \cos 2x + \cos (2x + x) \sin 2x$ ... (ii)

And

$\cos (x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$ ... (iii)

Now substituting equation (i) and (iii) in equation (ii), we get

$\Rightarrow \sin 5x = (\sin 2x \cos x + \cos 2x \sin x)\cos 2x + (\cos 2x \cos x - \sin 2x \sin x) \sin 2x$

$\Rightarrow \sin 5x = \sin 2x \cos 2x \cos x + \cos^2 2x \sin x + (\sin 2x \cos 2x \cos x - \sin^2 2x \sin x)$

$\Rightarrow \sin 5x = 2\sin 2x \cos 2x \cos x + \cos^2 2x \sin x - \sin^2 2x \sin x$ ... (iv)

Now $\sin 2x = 2\sin x \cos x$ ... (v)

And $\cos 2x = \cos^2 x - \sin^2 x$ ... (vi)

Substituting equation (v) and (vi) in equation (iv), we get

$\Rightarrow$ sin 5x = 2(2sin x cos x)(cos^{2}x - sin^{2}x)cos x + (cos^{2}x - sin^{2}x)^{2}sin x - (2sin x cos x)^{2}sin x

$\Rightarrow$ sin 5x = 4(sin x cos^2 x)([1 - sin^{2}x] - sin^{2}x) + ([1 - sin^{2}x] - sin^{2}x)^{2}sin x - (4sin^2 x cos^2 x)sin x (as cos^{2}x + sin^{2}x = 1 $\Rightarrow$ cos^{2}x = 1 - sin^{2}x)

$\Rightarrow$ sin 5x = 4(sin x [1 - sin^{2}x])(1 - 2sin^{2}x) + (1 - 2sin^{2}x)^{2}sin x - 4sin^3 x [1 - sin^{2}x]

$\Rightarrow$ sin 5x = 4sin x(1 - sin^{2}x)(1 - 2sin^{2}x) + (1 - 4sin^{2}x + 4sin^{4}x)sin x - 4sin^3 x + 4sin^{5}x

$\Rightarrow$ sin 5x = (4sin x - 4sin^{3}x)(1 - 2sin^{2}x) + sin x - 4sin^{3}x + 4sin^{5}x - 4sin^3 x + 4sin^{5}x

$\Rightarrow$ sin 5x = 4sin x - 8sin^{3}x - 4sin^{3}x + 8sin^{5}x + sin x - 8sin^{3}x + 8sin^{5}x

$\Rightarrow$ sin 5x = 5sin x - 20sin^{3}x + 16sin^{5}x

Hence LHS = RHS

Hence proved.

OR

We have to prove that $\cos\frac{\alpha-\beta}{2} = \frac{8}{\sqrt{65}}$

It is given that $\sin\alpha=\frac{4}{5}$ and $\cos\beta = \frac{5}{13}$,

We know,

$\sin^2\alpha + \cos^2\alpha = 1$

$\cos^2\alpha = 1 - \sin^2\alpha$

$\cos\alpha = \sqrt{1-\sin^2\alpha}$

$\cos\alpha = \sqrt{1-\left(\frac{4}{5}\right)^2} = \sqrt{1-\frac{16}{25}} = \sqrt{\frac{9}{25}}$

$\cos\alpha = \frac{3}{5}$

Similarly,

$\sin^2\beta + \cos^2\beta = 1$

$\sin^2\beta = 1 - \cos^2\beta$

sinb = $\sqrt{1-\cos^2\beta} = \sqrt{1-\left(\frac{5}{13}\right)^2} = \sqrt{1-\frac{25}{169}} = \sqrt{\frac{144}{169}}$

sinb = $\frac{12}{13}$

Identity used:

$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\cos(\alpha - \beta) = \frac{3}{5}\times\frac{5}{13}+\frac{4}{5}\times\frac{12}{13}$

$2\cos^2\left(\frac{\alpha-\beta}{2}\right) - 1 = \frac{15}{65} + \frac{48}{65}$

$2\cos^2\left(\frac{\alpha-\beta}{2}\right) = \frac{63}{65} + 1 = \frac{63+65}{65} = \frac{128}{65}$

$\cos^2\left(\frac{\alpha-\beta}{2}\right) = \frac{64}{65}$

$\cos\left(\frac{\alpha-\beta}{2}\right) = \sqrt{\frac{64}{65}}$

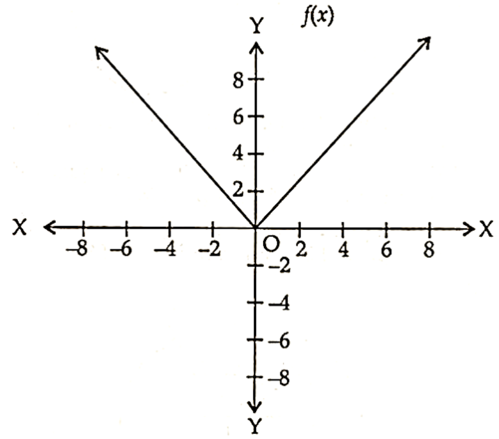
$\cos\left(\frac{\alpha-\beta}{2}\right) = \frac{8}{\sqrt{65}}$

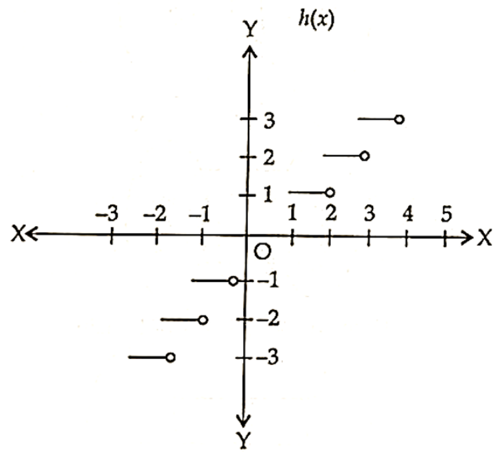
Hence Proved.

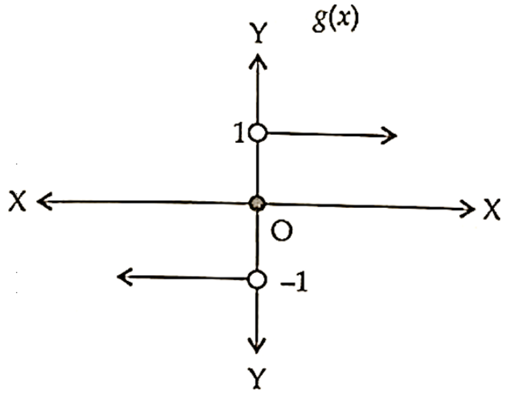
Section E

36. **Read the text carefully and answer the questions:**

Consider the graphs of the functions f(x), h(x) and g(x).







(i) h(x) = [x] is the greatest integer function. Its range is Z (set of integers)

(ii) f(x) = |x|. The domain of f(x) is R.

(iii)Since 10 > 0, f(10) = 1.

OR

g(x) is the signum function. Its range is {-1, 0, 1}.

37. **Read the text carefully and answer the questions:**

There are 4 red, 5 blue and 3 green marbles in a basket.

(i) Total marbles = 4 + 5 + 3 = 12

Required probability $= \frac{^4C_2}{^{12}C_2} = \frac{\frac{4\times3}{2\times1}}{\frac{12\times11}{2\times1}} = \frac{1}{11}$

(ii) Total marbles = 4 + 5 + 3 = 12

Required probability $= \frac{^3C_3}{^{12}C_3} = \frac{1}{\frac{12\times11\times10}{3\times2}} = \frac{1}{220}$

(iii)Total marbles = 4 + 5 + 3 = 12

Required probability $= \frac{^7C_2}{^{12}C_2} = \frac{\frac{7\times6}{2\times1}}{\frac{12\times11}{2\times1}} = \frac{21}{66} = \frac{7}{22}$

OR

Total marbles = 4 + 5 + 3 = 12

Required probability = 1 - P (None is blue)

$= 1 - \frac{^7C_3}{^{12}C_3}$

$= 1 - \frac{\frac{7\times6\times5}{3\times2}}{\frac{12\times11\times10}{3\times2}}$

$= 1 - \frac{7}{44} = \frac{37}{44}$

38. **Read the text carefully and answer the questions:**

Two complex numbers Z_1 = a + ib and Z_2 = c + id are said to be equal, if a = c and b = d.

(i) (x + iy)(2 - 3i) = 4 + i

$2x - (3x)i + (2y)i - 3yi^2 = 4 + i$

2x + 3y + (2y − 3x)i = 4 + i

Comparing the real & imaginary parts,

2x + 3y = 4 ...(i)

2y − 3x = 1 ...(ii)

Solving eq (i) & eq (ii), 4x + 6y = 8

−9x + 6y = 3

$13x = 5 \Rightarrow x = \frac{5}{13}$

$y = \frac{14}{13}$

$\therefore (x, y) = (\frac{5}{13}, \frac{14}{13})$

(ii) $x + iy = \frac{(1+i)^2}{2-i}$

$x + iy = \frac{(1+i)^2}{2-i} = \frac{1+2i+i^2}{2-i} = \frac{2i}{2-i} = \frac{2i(2+i)}{(2-i)(2+i)} = \frac{4i+2i^2}{4-i^2}$

$= \frac{4i-2}{4+1} = \frac{-2}{5} + \frac{4i}{5}$

$\Rightarrow x = \frac{-2}{5}, y = \frac{4}{5} \Rightarrow x + y = \frac{-2}{5} + \frac{4}{5} = \frac{2}{5}$